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Int. J. Heat Mass Transfer. Vol. 23, pp. 1690–1693
Pergamon Press Ltd. 1980. Printed in Great Britain

A NOTE ON MASS TRANSFER IN TURBULENT WALL JETS

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(*Received* 20 *August* 1979 *and in revised form* 1 *April* 1980)

NOMENCLATURE

a, exponent;
A, constant;
b, exponent;
B, constant;
c, mass concentration;
c_0, mass concentration at the surface of the porous plate;
d_0, width of nozzle (PWJ); diameter of nozzle (RWJ);
E, constant;
F, non-dimensional stream function;
G, non-dimensional concentration;
K, exchange coefficient;
u, mean velocity in the direction of flow;
U_M, maximum velocity (wall Jet);
U_0, nozzle exit velocity;
v, mean velocity in the transverse direction;
x, longitudinal coordinate;
x_0, distance of leading edge of porous plate from the virtual origin;
y, transverse coordinate;
y_n, height of nozzle above flat plate—radial wall jet;
δ, boundary layer thickness (for wall jets δ is the height at which $u = U_M/2$);
δ_c, concentration boundary layer thickness;
δ_M, height at which $u = U_M$;
η, similarity parameter (y/δ);
η_M, value of η at which $u = U_M$;
ζ, similarity parameter (y/δ_c);
u', v', c', turbulent fluctuations in u, v and c;
ρ, density;
ν, kinematic viscosity;
τ_0, wall shear stress;
τ, shear stress;
ε_M, diffusivity of momentum (turbulent);
ε_m, diffusivity of mass (turbulent);
Re_0, Reynolds number at nozzle exit $= U_0 d_0/\nu$;
Sc_t, turbulent Schmidt number.

1. INTRODUCTION

THE PROCESS of mass transfer from saturated porous surfaces exposed to turbulent air streams finds many practical applications. In many cases, the air stream will be in the form of a wall jet over the porous surface. The aerodynamics of both

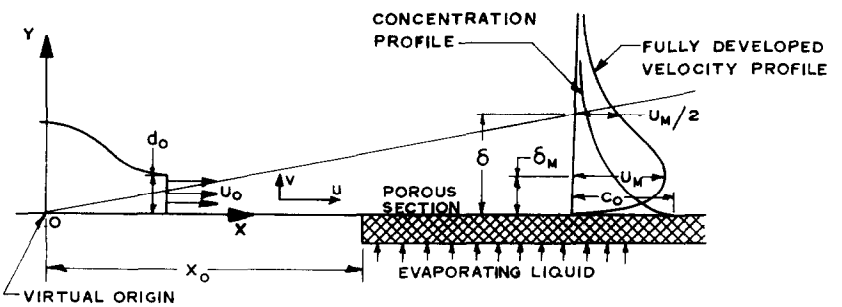

FIG. 1(a). Two-dimensional plane wall jet.

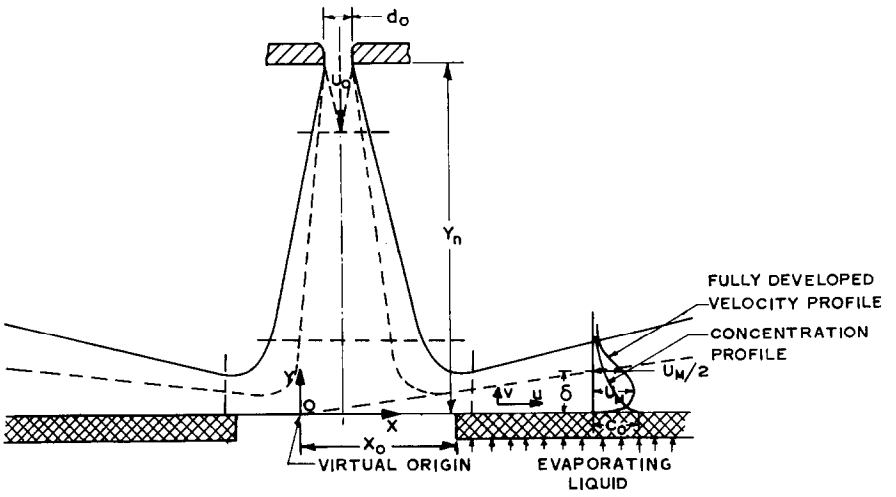

FIG. 1(b). Radial wall jet.

FIG. 1. Schematic representation of the plane and the radial wall jets with mass transfer.

plane and radial wall jets have been investigated in detail and a vast amount of literature is available on the subject [1–3]. On the other hand, mass transfer studies in turbulent wall jets are meagre. Mass transfer in both plane and radial wall jets, as shown schematically in Fig. 1, is considered herewith. 'Similarity solutions' to the governing equations are obtained and experimental evidence is presented which indicates that the concentration profiles are also self-preserving.

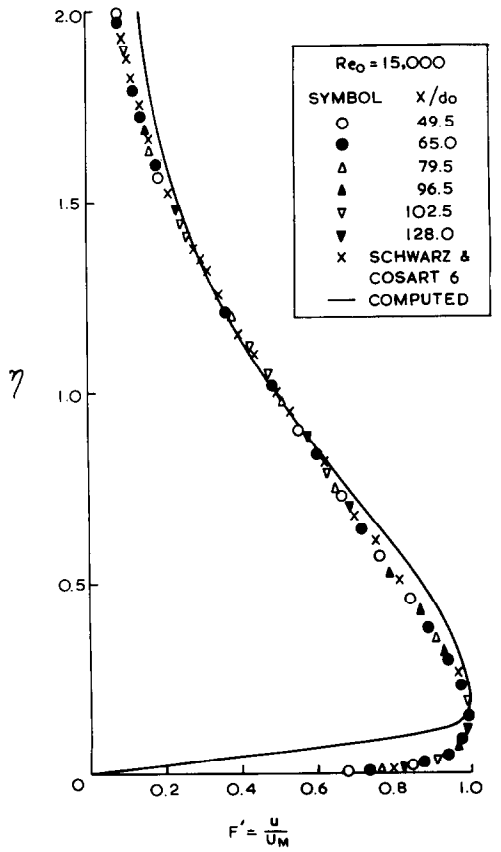

FIG. 2. Non-dimensional velocity profile. Two-dimensional plane wall jet.

2. THEORETICAL ANALYSIS

The governing equations for the flow under consideration (with reference to Fig. 1) are:

$$\text{Continuity} \qquad \frac{\partial}{\partial x}(ux^j) + \frac{\partial}{\partial y}(vx^j) = 0 \tag{1}$$

$$\text{Momentum} \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\tau}{\rho}\right) \tag{2}$$

$$\text{Conservation} \qquad u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = -\frac{\partial}{\partial y}(\mathring{m}'') \tag{3}$$

where

$j = 1$, for the radial wall jet

$j = 0$, for the plane wall jet

and

$$\frac{\tau}{\rho} = \left[\nu\frac{\partial u}{\partial y} - \overline{u'v'}\right]$$

$$\mathring{m}'' = \left[-D\frac{\partial c}{\partial y} + \overline{v'c'}\right].$$

Considering the momentum equation first, we seek a similarity solution for it by introducing the following:

1. Power law variation for the wall jet thickness δ and the maximum velocity U_m at any section given by $U_M = Ax^a$, $\delta = Bx^b$, x being measured from a 'virtual origin'.
2. A similarity parameter $\eta = y/\delta = y/Bx^b$
3. The self-preserving velocity profile $u/U_M = f(\eta)$
4. Representation of the shear stress as

$$\frac{\tau}{\rho} \approx -\overline{u'v'} = -U_M^2 h_1(\eta).$$

The continuity equation is eliminated by introducing the stream function ψ such that

$$u = \frac{1}{x^j}\frac{\partial \psi}{\partial y}; \quad v = -\frac{1}{x^j}\frac{\partial \psi}{\partial x}.$$

The stream function at any section x is then:

$$\psi = \int_0^y ux^j \,dy = ABx^{a+b+j} \int_0^\eta f(\eta)d\eta = ABx^{a+b+j}F(\eta).$$

With these substitutions the momentum equation (2) turns out as

$$A^2ax^{2a-1}F'^2 - A^2(a+b+j)x^{2a-1}FF'' = -\frac{A^2}{B}x^{2a-b}h_1'. \tag{4}$$

where, the primes designate differentiation with respect to η. The dimensional homogeneity of equation (4) implies that $b = 1$ both for the plane wall jet and the radial wall jet.

Consider now the momentum integral equation for these flows. This can be written as

$$\frac{d}{dx}\int_0^y u^2x^j \,dy + \frac{\tau_0}{\rho}x^j = 0$$

or on integration with respect to x,

$$\int_0^y u^2x^j \,dy + \int_0^x \frac{\tau_0}{\rho}x^j \,dx = \text{constant}. \tag{5}$$

It is now necessary to assume that the viscous dissipation term in the momentum integral equation is small compared to the total momentum of the jet. This assumption may be justified on the ground that the minimum value of the ratio of total momentum in the jet to the viscous dissipation at the wall as calculated from the experimentally measured profiles is of the order of 25 : 1 for the plane wall jet and 40 : 1 for the radial wall jet indicating a slow rate of momentum decay in wall jets [5]. Then equation (5) becomes

$$\int_0^y u^2x^j \,dy = \text{constant}$$

or

$$A^2Bx^{2a+b+j}\int_0^\infty F'^2 \,d\eta = \text{constant}. \tag{6}$$

The dimensional homogeneity of equation (6) implies $a = -\frac{1}{2}$ for the plane wall jet and $a = -1$ for the radial wall jet. It may be mentioned that the values of a and b thus obtained are in good agreement with those obtained from experimental data of other investigators [4].

With these values of a and b, the momentum equation reduces to

$$F'^2 + FF'' = \frac{(2-j)}{B}h_1'. \tag{7}$$

Next, consider the mass conservation equation. Here we may anticipate similarity of concentration profiles with respect to another parameter ζ defined by

$$\zeta = \frac{y}{E(x - x_0)}$$

where, E is a constant. Following the method used to transform the momentum equation we write the self preserving concentration profile as

$$c/c_0 = G(\zeta)$$

and

$$\dot{m}'' \approx \overline{v'c'} = U_M c_0 h_2(\zeta)$$

where, c_0 is the concentration of vapour along the surface of the plate (taken to be constant). The introduction of these into the mass conservation equation results in

$$\frac{FG^*}{(2-j)} = \frac{h_2^*}{B} \tag{8}$$

where * designates differentiation with respect to ζ. It can be easily shown that for a fixed value of x_0, the above equation can also be written as

$$\frac{FG'}{(2-j)} = \frac{h_2'}{B} \tag{9}$$

the primes, as before, denoting differentiation with respect to η.

Before, this set of non-linear ordinary differential equations (7) and (9) can be solved, a particular turbulent diffusion hypothesis has to be selected to connect h_1 and h_2 with F, G and their derivatives. There are many empirical hypothesis of turbulent transport none of which, however, is entirely satisfactory. For purposes of illustration, we choose the 'constant exchange coefficient' hypothesis (Prandtl) and write

$$\frac{\tau}{\rho} = \varepsilon_M \frac{\partial u}{\partial y} = K\delta U_M \frac{\partial u}{\partial y} \tag{10}$$

and

$$-\dot{m}'' = \varepsilon_m \frac{\partial c}{\partial y} = \frac{\varepsilon_M}{Sc_t}\frac{\partial c}{\partial y} = \frac{K\delta U_M}{Sc_t}\frac{\partial c}{\partial y}. \tag{11}$$

Substituting relations (10) and (11), equations (7) and (9) turn out as

$$(2-j)\frac{K}{B}F''' + FF'' + F'^2 = 0 \tag{12}$$

$$(2-j)\frac{K}{BSc_t}G'' + FG' = 0. \tag{13}$$

It is not possible to have a closed form solution of this set of non-linear ordinary differential equations with conditions

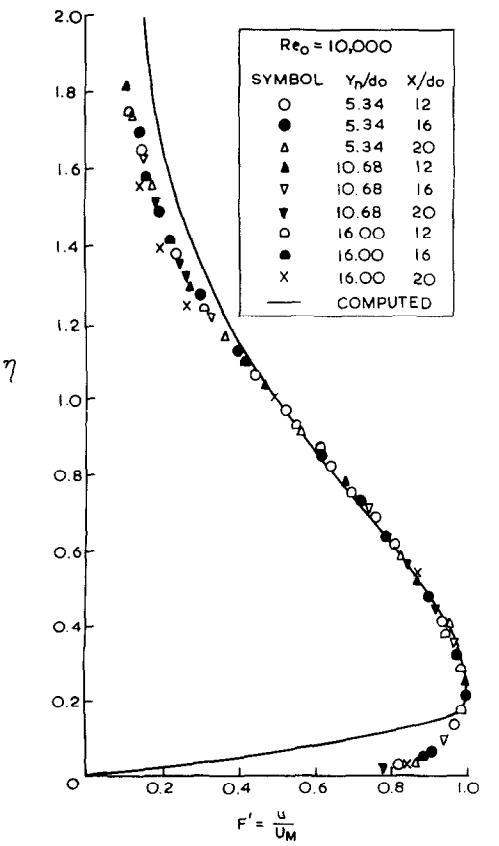

FIG. 3. Non-dimensional velocity profile. Radial wall jet.

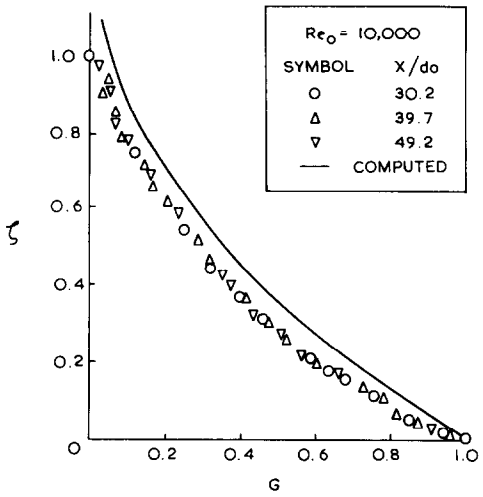

FIG. 4. Non-dimensional concentration profile. Plane wall jet.

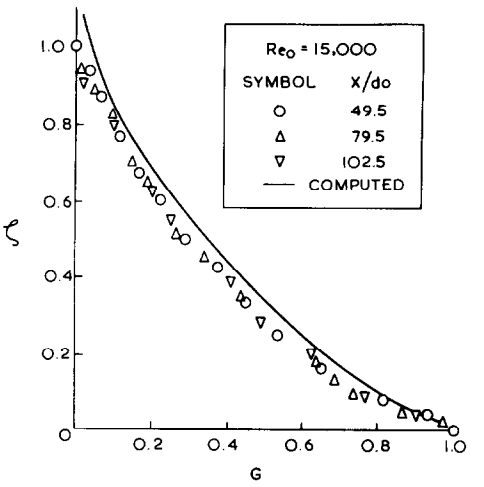

FIG. 5. Non-dimensional concentration distribution. Radial wall jet.

specified at two boundaries. Therefore, a numerical solution was obtained using the Runge–Kutta method.

Equations (12) and (13) can be written as

$$F''' = \frac{1}{\lambda}(FF'' + F'^2) \tag{14}$$

$$G'' = \frac{Sc_t}{\lambda}FG' = \frac{1}{\lambda}FG' \quad \text{(assuming } Sc_t = 1) \tag{15}$$

where λ = constant.

The momentum equation is independent of the mass conservation equation and can be solved by itself. In solving the momentum equation, the numerical integration was started by assuming $F = F' = 0$ at $\eta = 0$ (wall). Equating $F(0) = 0$, implies $v = 0$ at the wall. This is approximately true in the present case as the evaporation rates were very small. Typical values of $\mathring{m}''(0)$ and c_0 were 10^{-4} g cm^{-2} s and 20×10^{-4} g cm^{-3} giving value of $v = 0.05$ cm s^{-1} at the wall. The values of λ and $F''(0)$ were arrived at by trial and error to obtain the best fit of the computed curve of F' vs η with the experimental curve. The value of λ had to be changed in the outer layer again. For the plane wall jet, the values of λ selected were -0.011 in the inner layer and -0.41 in the outer layer. The corresponding values for the radial wall jet were -0.0158 and -0.36 respectively. These values of λ were then used in the integration of the mass conservation equation.

3. EXPERIMENTAL WORK

Two test rigs were fabricated; one for the plane wall jet and the other for the radial wall jet to simulate what is indicated in Figs. 1(a) and (b).

Smooth, homogeneously porous grade B 'Porosint' plates were used to evaporate methanol in the fully developed regions of the two turbulent wall jets. The concentrations of methanol at different distances and at different heights above the porous plates were measured by a gas chromatograph.

Mean velocity measurements in the wall jets were recorded by a narrow pilot tube coupled to a projection manometer. The fully developed mean velocity profiles in both cases exhibited 'similarity' and conformed quite closely to those given in literature.

Details of the experimental work can be found in [5].

4. RESULTS

The 'similar' velocity profiles of the plane wall jet and the radial wall jet are shown in Figs. 2 and 3. The computed profiles obtained by numerical integration of equation (12) are also indicated. The agreement between the computed profiles and the measured profiles is poor in the inner layer (between $y = 0$ and $y = \delta_M$) and improves in the outer region ($y > \delta_M$). This is mainly due to the poor representation of eddy viscosity near the wall by the constant exchange coefficient hypothesis.

Figures 4 and 5 show the measured and the computed non-dimensional concentration profiles in the plane wall jet and the radial wall jet respectively. The agreement between the two is reasonably good considering the crude hypothesis of turbulent transport assumed. It can be seen that the concentration profiles at different sections do exhibit 'similarity' in both cases with respect to the similarity parameter ζ as defined.

Acknowledgement—The author is grateful to Dr. M. Inderjit for help in the computation work and for providing the experimental data presented herewith.

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